LITERATURE CITED

1. A. N. Tikhonov and V. Ya. Arsenin, Methods of Solving Incorrect Problems [in Russian], Nauka, Moscow (1979).
2. O. M. Alifanov, "Regularization schemes to solve inverse problems of heat conduction," Inzh.-Fiz. Zh., 24, No. 2, 324-333 (1973).
3. A. N. Tikhonov, N. I. Kulik, I. N. Shklyarov, and V. B. Glasko, "On the results of mathematical modeling of a heat conduction process," Inzh.-Fiz. Zh., 39, No. 1, 5-10 (1980).
4. A. N. Tikhonov and V. B. Glasko, "On the question of methods of determining the temperature of a body surface," Vychisl. Mat. Mat. Fiz., 7, No. 4, 910-914 (1967).
5. V. B. Glasko, N. V. Zakharov, and A. Ya. Kolp, "On restoration of the thermal flux to a body surface for a nonlinear heat conduction process on the basis of the regularization method," Inzh.-Fiz. Zh., 29, No. 1, 60-62 (1975).
6. V. B. Glasko, G. V. Gushchin, and E. D. Mudretsova, "On utilization of drilling data for restoration of the contact shape by using the regularization method," Vychisl. Mat. Mat. Fiz., 14, No. 5, 1272-1280 (1974).
7. V. A. Ditkin and A. P. Prudnikov, Operational Calculus [in Russian], Vysshaya Shkola, Moscow (1975).
8. V. A. Morozov, "On the residual principle for solving operator equations by the regularization method," Zh. Vyshisl. Mat. Mat. Fiz., 8, No. 2, 295-305 (1968).
9. A. N. Tikhonov, V. B. Glasko, and N. I. Kulik, "Regularizing algorithms for nonlinear problems and inverse problem of magnetotelluring sounding," Computation Methods and Programming [in Russian], No. 20, Moscow Univ. Press (1973), pp. 158-174.
10. S. M. Nikol'skii, Approximation of Functions of Many Variables and Imbedding Theorems [in Russian], Nauka, Moscow (1969).
11. A. N. Kolmogorov and S. V. Fomin, Elements of the Theory of Functions and Functional Analysis, Graylock (1961).
12. V. S. Vladimirov, Generalized Functions [in Russian], Nauka, Moscow (1976).
13. A. N. Tikhonov, V. B. Glasko, and O. K. Litvinenko, "On continuation of a potential towards perturbing masses on the basis of the regularization method," Fiz. Zemli, No. 12, 30-48 (1968).

ANALYSIS OF TEMPERATURE FIELDS OF BODIES IN THE SHAPE OF SHELLS

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The temperature fields in axisymmetric thick-walled shells with different middle surface shape are investigated.

In performing thermal engineering analyses of metallurgical or power equipment there often occurs the need to solve heat conduction problems for bodies in the shape of shells. In these cases it is natural to use shell theory methods [1-3]. The nonclassical theory of shells [2], whose equations are valid for nonthin shells with rapidly varying geometry and thickness, is used below to determine the nonstationary temperature field of a unified slag car cup.

A slag car cup is an axisymmetric thick-walled shell formed by a spherical segment connected to a hollow truncated cone of linearly varying thickness; hence, the boundary conditions will also be axisymmetric, which permits solution of the problem for the domain displayed in Fig. 1. The slag pouring periods and further heating of the cup up to the time of emptying are considered; it is assumed in the computations that the thermophysical constants of the cup material are independent of the temperature.

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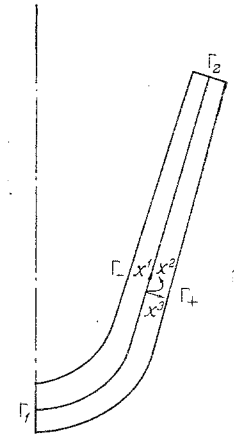

Fig. 1. Computation domain.

Taking account of the boundary-value problem presented above for the nonstationary heat conduction equation in orthogonal curvilinear coordinates, we can write the following [2]:

$$\frac{1}{\sqrt{g}}\frac{\partial \sqrt{g}\,q^1}{\partial x^1}+\frac{1}{\sqrt{g}}\frac{\partial \sqrt{g}\,q^3}{\partial x^3}+c\rho\frac{\partial T}{\partial t}=0,$$
$$T=T_0 \text{ at } t=0;$$
$$q^3|_{\Gamma_-}=\alpha_-(\Theta_- - T|_{\Gamma_-});\tag{1}$$
$$q^3|_{\Gamma_+}=\alpha_+(T|_{\Gamma_+}-\Theta_+);$$
$$q^1|_{\Gamma_1}=0;\quad q^1|_{\Gamma_2}=\alpha_2(T|_{\Gamma_2}-\Theta_+).$$

We seek the shell temperature in the form of Legendre polynomials of degree N in the variable x^3 ($-h\leqslant x^3\leqslant h$) [1, 2]

$$T=\sum_{n=0}^{N}\overset{n}{T}(t,\ x^1)\,h^{-n-1}P_n\left(\frac{x^3}{h}\right),$$

where $2h(x^1)$ is the shell thickness, $P_n(x^3/h)$ are Legendre polynomials that are orthogonal in the segment $[-h,\ h]$, $\overset{n}{T}(t,\ x^1)=\left(n+\frac{1}{2}\right)h^n\int_{-h}^{h}TP_n(x^3/h)\,dx^3$ are the temperature moments determined from the system of partial differential equations obtained by applying the projection method [2] to (1).

The following values of the parameters are taken in the computations: $c = 670.4$ J/(kg•°K), $\rho = 7850$ kg/m^3, $T_0 = 293$°K, $\lambda = 32.6$ W/(m•°K), $\Theta_+ = 293$°K

$$\Theta_-=\begin{cases}\Theta_+ \text{ — inner surface washed by air,}\\ T_p \text{ — inner surface in contact with slag,}\end{cases}$$

The slag temperature was calculated from the formula

$$T_p=\frac{T_2-T_1}{\Delta t}t+T_1,$$

where $T_1 = 1473$°K, $T_2 = 1323$°K $\Delta t = 120$ min; α_+, α_2, α_- for the sections washed over by air were calculated from the formula

$$\alpha=\alpha_l+4.77\cdot 10^{-8}\,\frac{T_c^4-\Theta_+^4}{T_c-\Theta_+},$$

where T_c is the temperature on the surface, $\alpha_l = 9.31$ W/(m^2•°K). For the section of the surface washed over by the slag

$$\alpha_-=\frac{\beta_2-\beta_1}{\Delta t}t+\beta_1,$$

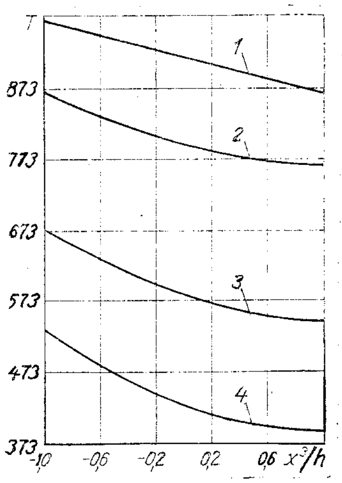

Fig. 2. Distribution of the temperature T (°K) over the dimensionless wall thickness (x^3/h) corresponding to the juncture of the spherical and conical parts of the cup: 1) 15; 2) 30; 3) 60; 4) 120 min.

where β_1 = 98.93 W/(m^2•°K), β_2 = 86.07 W/(m^2•°K).

The system of differential equations for $\overset{n}{T}$ (n = 0, 1, ..., N) was written in the vector form

$$\frac{\partial \mathbf{T}}{\partial t} + \|A\| \frac{\partial^2 \mathbf{T}}{\partial (x^1)^2} + \|B\| \frac{\partial \mathbf{T}}{\partial x^1} + \|C\| \mathbf{T} + \mathbf{D} = 0, \qquad (2)$$

where $||A||$, $||B||$, $||C||$ are matrices of dimensionality (N + 1) × (N + 1), D is a vector of dimensionality N + 1 whose elements are formed directly in the FORTRAN program. Equation (2) was solved by the matrix factorization method [4]. The structure of the program permits solution of the nonstationary heat conduction problem for axisymmetric thick-walled shells with different middle surface shapes by inducing insignificant changes. The time to compute the temperature change per time spacing is 34.5 sec on the ES-1050 computer for N = 3.

Investigations by the method of a numerical experiment indicate a rise in the rate of convergence of the results obtained as the dimensionality of the coordinate base increases. If solutions with dimensionalities N = 0 and N = 1 are unacceptable, then it is sufficient to take N = 3 to obtain a practically exact result; the divergence of the solution is less than 1% for $N \geqslant 4$.

The temperature distribution over the cup wall thickness is shown in Fig. 2 for a section corresponding to the juncture of the sphere with the cone. It is established that during the pouring of the slag into the cup (the duration of the slag pouring period is taken at 20 min) the temperature change over the wall thickness is substantially nonlinear in nature; at the end of the pouring period the transverse gradients reach the greatest values (to 15 deg/cm), then a gradual equalization of the temperature occurs with the lapse of time, and its distribution over the wall becomes linear.

Application of nonclassical shell theory [2] permitted computation of the temperature fields not only for the spherical—conical cup being exploited but also the execution of a comparative thermal analysis of paraboloidal and ellipsoidal shells in which shapes slag cups can be made. The geometric parameters of the middle surfaces of the shells mentioned were selected in such a way that they would satisfy the main requirements imposed on the spherical—conical cups used at this time. The quantity of mesh nodes on the meridians of the paraboloidal and ellipsoidal shells, as well as their mutual location corresponded to the quantity and location of the nodes for the spherical—conical shell (a certain difference is explained by the fact that the lengths of the shell meridians differ somewhat), which permitted a comparative analysis of the results.

Changes in the mean temperature gradient T/2h with respect to the wall thickness are shown in Fig. 3 for two sections (points 8 and 15 on the shell meridians), the first of which corresponds to the juncture of the sphere and the cone in the sphere—cone cup, and the second

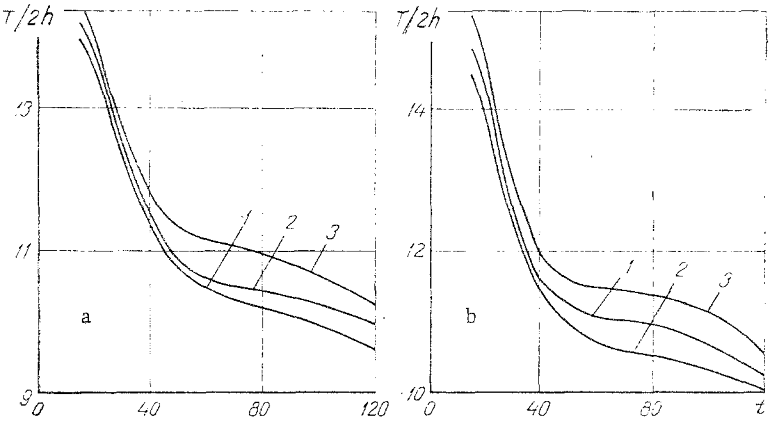

Fig. 3. Change in the mean temperature gradient T/2h (deg/cm) with respect to the wall thickness as a function of the time t (min) for points 8 (a) and 15 (b) on the shell meridians: 1) sphere–cone; 2) ellipsoidal; 3) paraboloidal.

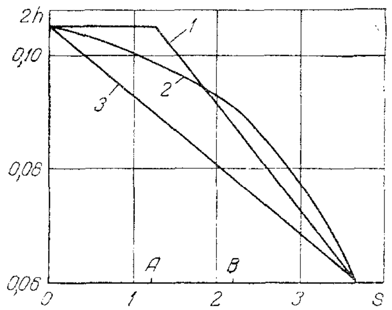

Fig. 4. Change in the shell thickness 2h (m) along the meridians (s is the running length of the arc of a meridian), m): 1) sphere–cone; 2) ellipsoidal; 3) paraboloidal shell. Values of the functions at points A and B correspond to the shell thicknesses at the points 8 and 15.

is perpendicular to the meridian at a point approximately at its middle (approximately 29 finite-difference divisions were superposed on the shell meridian).

The greatest temperature gradients at point 8 during the whole time period under consideration were observed in the paraboloidal shell, somewhat smaller ones in the ellipsoidal, and still smaller in the sphere–cone shell. The pattern changes somewhat at point 15. As before, the greatest gradients are in the paraboloidal shell, however, the next gradients in magnitude are in the sphere–cone, and the smallest in the ellipsoidal shell.

It is seen from Fig. 4 on which the change in shell thickness along their meridians is displayed (s is the running length of the meridian) that at the points being considered the paraboloidal shell has the least thickness, which is indeed the reason for the greatest gradients. At point 8 the sphere–cone shell is thicker than the ellipsoidal; hence, the gradients in this section are less for it. The shell thickness changes with progress along their meridians. At point 15 the ellipsoidal cup becomes thicker than the sphere–cone; consequently, larger gradients occur in the sphere–cone shell. Therefore, the law of thickness variation exerts the governing influence on the magnitude of the temperature gradients in sections normal to the shell middle surface.

Taking account of the closeness of the results obtained for the shells examined, the law of thickness variation for a given weight can be selected for the paraboloidal and ellipsoidal shells so that the thermal loads acting on them would not exceed the loads on the sphere–cone shell. In that case the paraboloidal and ellipsoidal shells are preferable to the sphere–cone since they have a convexity opposing the normal displacements to the middle surface caused by the thermal loads and a continuous change in the principal radius of curvature along the meridian.

NOTATION

T, temperature; x^1, x^3, coordinate system axes coupled normally to the shell middle surface; q^1, q^3, contravariant components of the thermal flux density vector in the x^1 and x^3 directions, respectively; g, determinant of the metric form; c, specific heat; ρ, density; T_0, initial temperature; Θ_+, air temperature; Θ_-, temperature of the medium washing over the shell at the boundary Γ_-; α_+, α_2, α_-, heat elimination coefficients to the boundaries Γ_+, Γ_2, Γ_-; Δt, time period between the beginning of slag pouring and the time of its termination.

LITERATURE CITED

1. I. N. Vekua, Theory of Thin, Variable-Thickness Shallow Shells [in Russian], Metsniereba, Tbilisi (1965).
2. V. I. Gulyaev, V. A. Bazhenov, and P. P. Lizunov, Nonclassical Shell Theory and Its Application to the Solution of Engineering Problems [in Russian], Vishcha Shkola, Lvov (1978).
3. Ya. S. Podstrigach and R. N. Shvets, Thermoelasticity of Thin Shells [in Russian], Naukova Dumka, Kiev (1978).
4. A. A. Samarskii and E. S. Nikolaev, Methods of Solving Mesh Equations [in Russian], Nauka, Moscow (1978).

SOLUTION OF THE INVERSE COEFFICIENT PROBLEM OF HEAT CONDUCTION

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An effective algorithm is developed for solving the inverse coefficient problem of heat conduction. Methods are proposed for improving the nonuniform convergence of the solution at the domain boundaries.

In the past decade the problem of determining the thermophysical characteristics of materials in nonstationary heating regimes has attracted the considerable attention of researchers. This is related primarily to the impossibility of determining the mentioned characteristics by classical methods in a number of cases of practical importance, for example, for coking materials, as well as to the possibilities being disclosed here of a substantial rise in the productivity and simplification of experimental investigations. However, serious difficulties associated with the necessity to solve inverse coefficient problems of heat conduction occur here. The development of methods to solve these problems was studied in [1-10].

In conformity with [1, 2], and using the method proposed in [11] to calculate the gradient of the functional of the quadratic residual of experimental and computed temperatures, an effective algorithm to solve the inverse coefficient problem of heat conduction is given in this investigation which would assure good accuracy in determining the heat conduction coefficient λ(T) and the specific heat c(T).

Let us consider the solution of the inverse problem for an infinite plate. In order not to limit the possibilities of the experiment we consider that boundary conditions of the first, second, and third kinds can be given on the plate surfaces. The temperature distribution in the plate will hence be described by the equation

$$c(T)\rho \frac{\partial T}{\partial t} = \frac{\partial T}{\partial x}\left(\lambda(T)\frac{\partial T}{\partial x}\right),\ 0<x<\delta,\ 0<t\leqslant t_e \tag{1}$$

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